Exam
SOLID MECHANICS (NASM)
January 20, 2014, 14:00–17:00 h

Question 1 For each of the following statements point out if it is correct or not, *and* why.

a. The normal stress component parallel to a surface can be prescribed by the tangential component of a traction boundary condition. (For example, consider a surface span by base vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$, with unit normal $\boldsymbol{e}_3$; then the statement implies that a boundary condition for the traction component t_1 prescribes the normal stress σ_{11}.)

b. Plastic slip in a material with isotropic linear elasticity is volume-preserving only when the Poisson ratio $\nu = 1/2$.

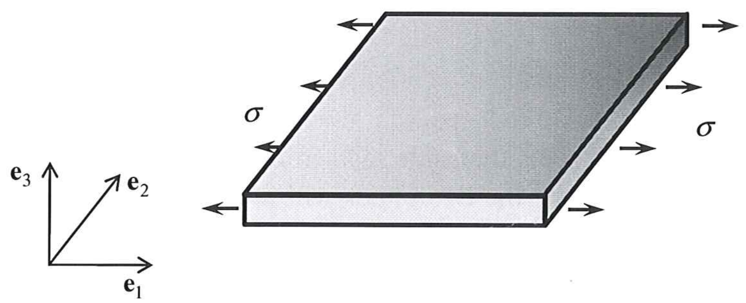
c. The small-strain solution (i.e., using geometrically linear theory) for stress ($\boldsymbol{\sigma}$), strain ($\boldsymbol{\varepsilon}$) and displacement ($\boldsymbol{u}$) in *every* material has to satisfy the equations:

$$\operatorname{div} \boldsymbol{\sigma} = \boldsymbol{0}, \tag{1a}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}[\operatorname{grad} \boldsymbol{u} + (\operatorname{grad} \boldsymbol{u})^T], \tag{1b}$$

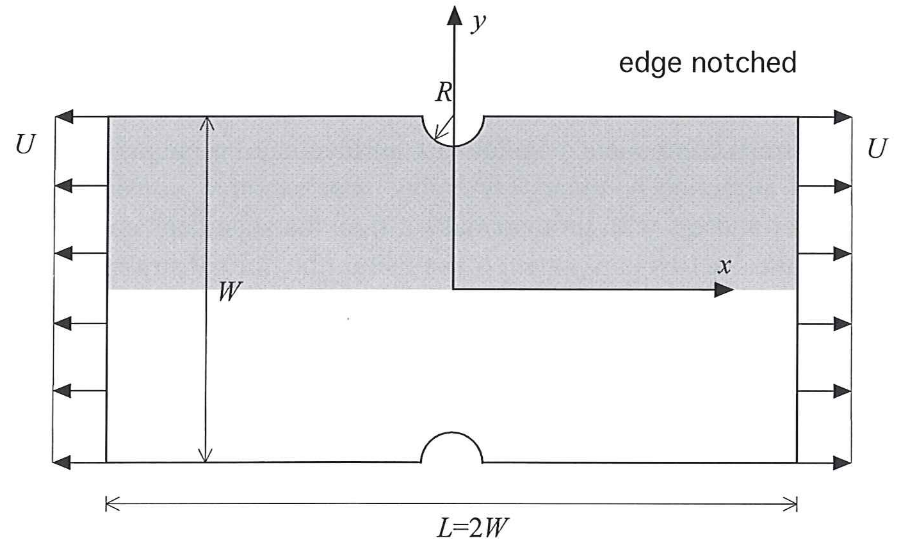
$$\boldsymbol{\sigma} = \mathscr{L}\boldsymbol{\varepsilon}. \tag{1c}$$

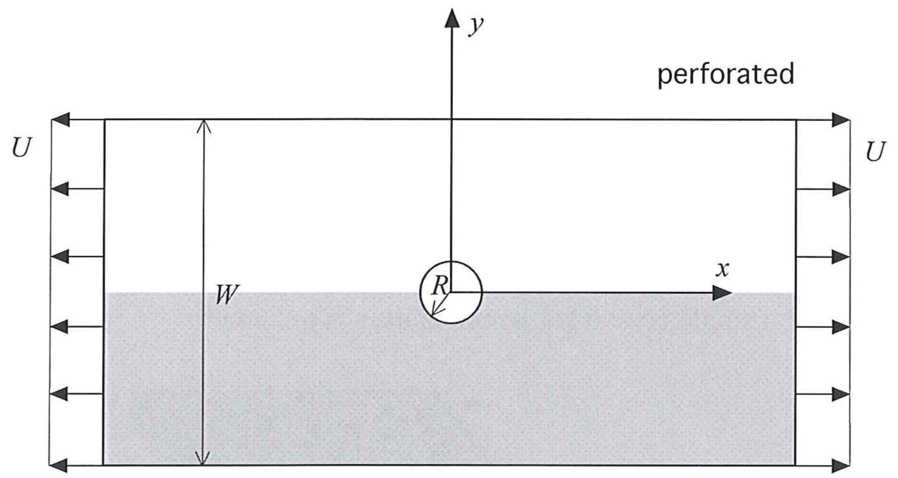
Question 2 Consider a homogeneous plate subject to tension in the $\boldsymbol{e}_1$ direction, see figure. The plate is made of a single crystal but its orientation is unknown.



Assume a uniaxial stress state (i.e. plane stress in both $\boldsymbol{e}_2$- and $\boldsymbol{e}_3$- directions). Determine the most likely slip plane(s) and slip direction(s), i.e., the one(s) where the resolved shear stress is maximum.

Question 3 Two rectangular plates of W by $2W$ are perforated with a hole of radius R but in different ways: one with two half-circular notches at the edges, the other with a central hole, as shown in the figure below.

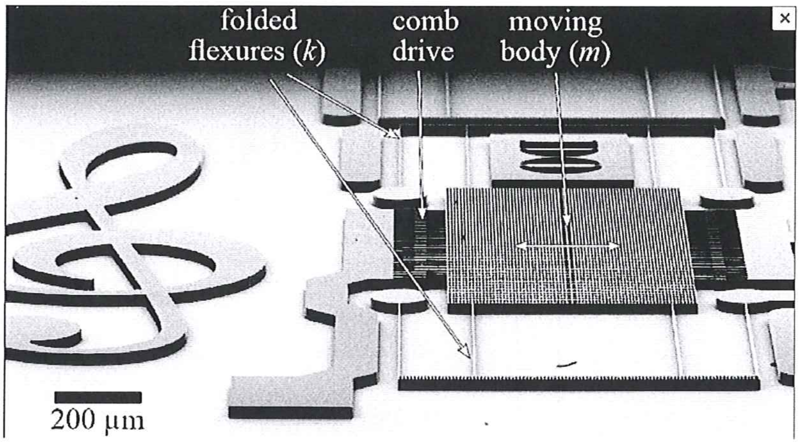




The total amount of material left is the same, and in fact the gray areas indicating symmetric halves are identical. Nevertheless, the two plates will respond to an applied end displacement U in different ways (even if they have the same uniform material properties). Apparently something in the entire set of governing equations is different; the question is: what?

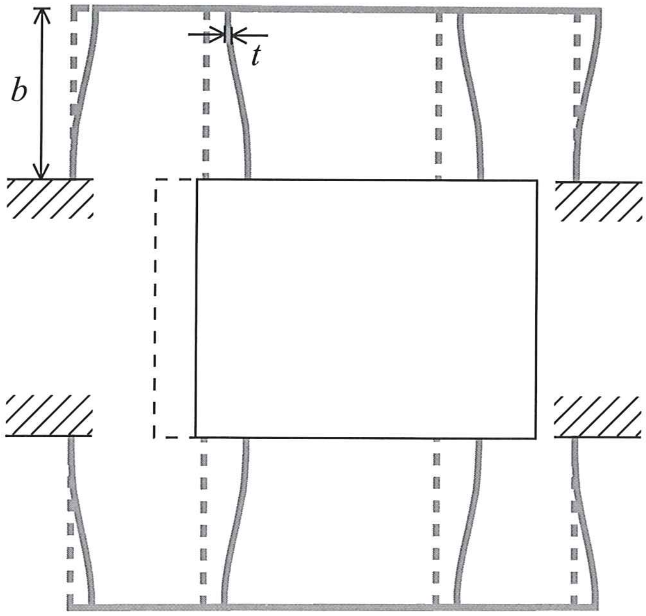
Substantiate your explanation with mathematical statements.

Question 4

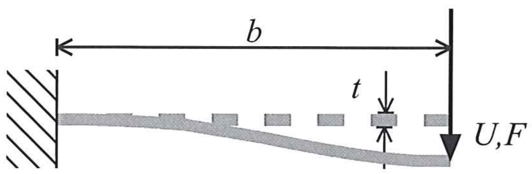


A group of nanotechnology students from Twente University has developed a musical instrument called "micronium", which is able to produce tones with audible frequencies, despite the fact that the vibrating mass weighs only a few dozen micrograms. This is non-trivial in view of the well-known formula $f = \sqrt{k/m}/(2\pi)$ for the eigenfrequency of a mass (m)–spring system (stiffness k): at small scales, the mass m is small so that the stiffness needs to be small in order to keep f in the audible range ($\sim$ 20–20.000 Hz). The device has a separate unit for each tone; a scanning electron microscopy image of one unit is shown above. It consists of a thin rectangular plate that is mounted in between spring structures at the top and the bottom consisting of thin slender beams (the comb structures on the left and the right-hand side serve to actuate ('pluck') the mass-spring system and as a sensor to pick up of the frequency).

The figure below shows a mechanical model, comprising four interconnected leaf springs on top and on bottom, each having length b, thickness t and width w. Both the equilibrium position (dashed lines) and a deflected configuration are shown (the outer leaf springs are mounted on posts fixed to the wafer).



a. First determine the stiffness $k_1 := F/U$ of a *single* leaf spring against a sideway displacement U, see figure below (this can be done either by solving the differential beam equation (3.49) plus boundary conditions or by application of the vergeet-me-nietjes of Fig. 3.6).



b. Given the stiffness k_1 of a single leaflet, what is the total stiffness k of the spring system in a micronium unit?

Question	# points
1	3
2	5
3	3
4	4+2=6

Exam grade = (# points + 2)/1.9

Opgave 1

Opgave 2

Stress state (wrt given basis): $\sigma_{11} = 0$, $\sigma_{ij} = 0$ otherwise. Has principal stresses $\sigma_1 = \sigma_{22}$ and $\sigma_2 = \sigma_3 = 0$. [1]

Hence maximum shear stress occurs under 45°

in the $\boldsymbol{e}_1$–$\boldsymbol{e}_2$ plane [1]

as well as in the $\boldsymbol{e}_1$–$\boldsymbol{e}_3$ plane. [1.5]

Opgave 3 [1.5]

Only difference is in BCs along $y = 0$. Holes are always traction free. [1]

Differences:

top edge notched: $t_y(x, W/2) = 0$; central hole: $u_y(x, 0) = 0$ [1]

btm edge notched: $u_y(x, 0) = 0$; central hole: $t_y(x, -W/2) = 0$ [1]

Opgave 4

a. Use symmetry. The deflection half way, i.e. $U/2$, is the same as that of a cantilever of length $b/2$ loaded by an end load F; that is: [1]

$$\frac{U}{2} = \frac{(F)(b/2)^3}{3EI} = \frac{Fb^3}{24EI} \Longrightarrow U = \frac{Fb^3}{12EI}$$

Hence [2]

$$k_1 := \frac{F}{U} = \frac{12EI}{b^3},$$

or with $I = Wt^3/12$,

$$k_1 := \frac{EWt^3}{b^3},$$

[1]

b. A doublet of leaflets acts springs in series, so $k_2 = k_1/2$. There are two doublets on top and two on the bottom. The two doublets on top are in parallel, so that $k_4 = 2k_2 = k_1$. The four leaflets on top and those on the bottom are in parallel again, so [1]

$$k = 2k_4 = 2k_1 = \frac{24EI}{b^3} = \frac{2EWt^3}{b^3}.$$

[1]

HEI

$\frac{1}{2} ld^2 - \frac{1}{6} d^3$

$d^2 \left(\frac{1}{2} L - \frac{1}{6} d\right)$

$\frac{1}{2} d^2 \left(L - \frac{1}{3} d\right)$

$\frac{3}{6} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

$\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

$\frac{2 h L^3}{3} \cdot \frac{1}{d^2 (L - \frac{1}{3} d)}$

$\frac{F}{EI} \left(\frac{1}{3} L^3 - \frac{1}{2} L^2 d + \frac{1}{3} L d^2\right)$

$\frac{F}{3EI} \left(L^3 - \frac{3}{2} L^2 d + L d^2\right)$

$\frac{3EI}{L^3 - \frac{3}{2} L^2 d + L d^2}$

$\frac{1}{2} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{1}{3}$

$\frac{2}{3} \frac{h l^3}{d^2 (L - \frac{2}{3} d)} = \frac{F}{EI} \left(\frac{1}{2} L d^2 - \frac{1}{6}\right.$

$\omega(l) = \frac{F}{EI} \left(\frac{1}{2} L^3 - \frac{1}{6} L^3\right) = \frac{F}{EI} \frac{1}{3} L^3$

$\frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$

$\frac{F}{\frac{F L^3}{3EI}} = \frac{3EI}{L^3}$

$\frac{F}{\frac{F L (\frac{1}{6} l^3 - L d + \frac{1}{3} d^3)}{2EI}}$

$\frac{\frac{1}{2}}{\frac{1}{3}} =$

$\frac{1}{6} \cdot 8 - 4 + \frac{2}{3}$

$\frac{2EI}{\frac{1}{6} L^3 - L^2 d + \frac{1}{3} l d^3}$

$\frac{8}{6} - \frac{24}{6} + \frac{4}{6} \qquad \frac{8}{6} - \frac{20}{6}$

$\frac{2}{1} = \frac{3}{\frac{2}{3}}$

$\frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$

$-\frac{2}{2} \qquad -\frac{12}{6}$

$\frac{3}{8} \qquad \frac{6}{1{,}2} =$

$5 \cdot 5 \cdot 1{,}2 =$

0,55

1,1 $\cdot$ 5,5

6,6

$\frac{1}{6}$ vs $\frac{1}{5}$